ANSWER KEY 54-51

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2020

PART III MATHEMATICS (COMMERCE)

Code: 54-51 VERSION: R

MARKS: 80 TIME: 2½ Hrs.

Q. No.	Sub Qns	Answer key / Value Points	Score	Total Score
1	(i)	$a_{11}=0$, $a_{12}=-1$, $a_{21}=1$, $a_{22}=0$	1½	
		$A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$	½	3
	(ii)	$A'=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$	1	
		Remarks: $A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$, give ½ score		
2	(i)	(b) $\frac{\pi}{6}$	1	
	(ii)	(a) $\frac{\pi}{2}$	1	3
		(b) π	1	
		Remarks: 1) For $\sin^{-1}(\frac{1}{2})=\frac{\pi}{6}$, $\cos^{-1}(\frac{1}{2})=\frac{\pi}{3}$, give ½ score 2) For the formula $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$, give ½ score 3) For the formula $\cos^{-1}(-x)=\pi-\cos^{-1}x$, give ½ score		
3	(i)	$\lvert\bar{a}\rvert=\sqrt{2^2+1^2+(-1)^2}=\sqrt{6}$	½	
		$\lvert\bar{b}\rvert=\sqrt{1^2+2^2+1^2}=\sqrt{6}$	½	
	(ii)	$\bar{a}\cdot\bar{b}=2\times1+1\times2+-1\times1=3$	1	3
	(iii)	$\cos\theta=\frac{3}{\sqrt{6}\times\sqrt{6}}=\frac{1}{2}$	1	
		$\therefore\theta=\frac{\pi}{3}$		
		Remarks: 1) For writting formula for $\lvert\bar{a}\rvert$, give ½ score 2) For writting formula for $\bar{a}\cdot\bar{b}$, give ½ score 3) For writting formula for $\cos\theta=\frac{\bar{a}\cdot\bar{b}}{\lvert\bar{a}\rvert\cdot\lvert\bar{b}\rvert}$, give ½ score 4) Full score may be given for finding upto $\cos\theta=\frac{1}{2}$		

Qn No	Sub Qn.	Answer Key / Value Points	Score	Total score
4	(i)	$x = \tan\frac{\pi}{4}$	1	
		$= 1$		
	(ii)	$LHS = \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right)$	1	3
		$= \tan^{-1}\left(\frac{125}{250}\right) = \tan^{-1}\frac{1}{2}$	1	
		Remarks: For writting formula for $\tan^{-1}x + \tan^{-1}y$, give 1 score.		
5.		Marginal cost $= c'(x)$	1	
		$= 24x + 5$	1	3
		Put $x = 500$, $c'(500) = 24\times500+5 = 12005$	1	
		Remarks: For writting $c'(500)$ only, give $\frac{1}{2}$ score		
6.	(i)	$\int\frac{1}{1+x^2}dx = \tan^{-1}x + c$	1	
	(ii)	Put $t = \tan^{-1}x$, then	$\frac{1}{2}$	
		$dt = \frac{dx}{1+x^2}$	$\frac{1}{2}$	3
		$\therefore \int\frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt$	$\frac{1}{2}$	
		$= e^t + c = e^{\tan^{-1}x} + c$	$\frac{1}{2}$	
		Remarks: Full marks may be given even if 'c' is not written in answer		
7	(i)	(a) one	1	
	(ii)	$\bar{r} = (i+2j+k) + \lambda(2i+3j+k)$	1	3
		and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-1}{1}$	1	
		Remarks: $\frac{1}{2}$ scores each may be given for each formula.		
8.		Getting the number 5 is a success	$\frac{1}{2}$	
		$p = P(\text{success}) = \frac{1}{6}$ $\quad q = P(\text{failure}) = \frac{5}{6}$	$\frac{1}{2}+\frac{1}{2}$	3
		Required probability $= {}^nC_r\, q^{n-r}\, p^r$	$\frac{1}{2}$	
		$= {}^7C_2\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)^2$	1	
		$= \frac{21\times5^5}{6^7}$		

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9.	(i)	$A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$	1/2	
		$A+A' = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$	1/2	
		$A-A' = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$	1/2	
		$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$	1/2	4
	(ii)	Since $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is skew-symmetric,		
		$a = d = 0$	1/2	
		$b = -c$	1/2	
		$\therefore \frac{a+b}{c+d} = \frac{0+-c}{c+0}$.	1/2	
		$= \frac{-c}{c} = 1$	1/2	
		Remarks: Give 1 score for $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$		
10	(i)	$LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2} kx^2 = 4k$	1/2	
		$RHL = \lim_{x \to 2+} f(x) = \lim_{x \to 2} 3 = 3$	1/2	
		Since $f(x)$ is continuous, $4k = 3$	1/2	
		$\therefore k = 3/4$	1/2	4
	(ii)	Let $f_1(x) = \cos x$, $f_2(x) = x^2$		
		Then f_1 is continuous, being a trigonometric functn	1/2	
		f_2 is continuous, being a polynomial function	1/2	
		$\therefore$ Their composite function $f_1 \circ f_2(x) = \cos(x^2)$ is also continuous	1	
		Remarks: For $LHL = RHL$, concept of continuous function, give 1 score.		

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11	(i)	$f(3) = 3\times3^2+3-3 = 27$	1	
	(ii)	$f'(x) = 6x+1$	1	4
		$f'(3)\Delta x = 19\times 0.02 = 0.38$	½	
		$f(3.02) = f(3) + f'(3)\cdot\Delta x$	½	
		$= 27 + 0.38 = 27.38$	1	
		Remarks: In (ii), for writting formula, give 1 score.		
12	(i)	(c) −1	1	
	(ii)	$\int_0^5 e^x dx = \left[e^x\right]_0^5 = e^5 - 1$	1	
	(iii)	Put $f(x) = \sin x \quad \therefore f'(x) = \cos x$	½	4
		$\therefore I = \int_0^{\pi/4} e^x\left(f(x)+f'(x)\right)dx$	½	
		$= \left[e^x f(x)\right]_0^{\pi/4}$	½	
		$= \left[e^x \sin x\right]_0^{\pi/4} = e^{\pi/4}\sin\frac{\pi}{4} = \frac{e^{\pi/4}}{\sqrt{2}}$	½	
		Remarks: For writting formula, give ½ score		
13	(i)	$f'(x) = 3x^2 - 3$	1	
	(ii)	$f(x)$ is continuous in $[-1, 1]$	½	
		$f(x)$ is differentiable in $(-1, 1)$	½	
		$f(1) = 1, \quad f(-1) = 5$		
		$f'(c) = \frac{1-5}{1-(-1)} = -2$	½	4
		ie, $3c^2 - 3 = -2 \quad \therefore c^2 = \frac{1}{3} \quad c = \pm 1/\sqrt{3}$	½	
		$\therefore$ [illegible] $\therefore c \in (-1, 1)$. MVT is verified		
	(iii)	$f'(c) = 0$ means $3c^2 - 3 = 0$	1	
		$\therefore c = \pm 1$		

Q. No	Sub Q	Answer Key / Value Points	Score	Total score
14		The equation of the circle is $x^2+y^2=4$	½	
		The point of intersection of line and circle is $(\sqrt{2}, \sqrt{2})$	½	
		Area under the line $= \int_0^{\sqrt{2}} x\,dx = 1$	1	4
		Area under the circle $= \int_{\sqrt{2}}^{2} \sqrt{4-x^2}\,dx$.	½	
		$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{2}}^{2}$	1	
		$= \frac{\pi}{2} - 1$		
		$\therefore$ Total area $= 1 + \left(\frac{\pi}{2}-1\right) = \frac{\pi}{2}$	½	
		Remarks: 1) For formula $\int_a^b y\,dx$, give ½ score 2) For the concept Area = definite integral, give 1 score		
15	(i)	Order = 1, degree = 1	½+½ = 1	
	(ii)	$\frac{dy}{dx} - \frac{y}{x} = 1$	½	4
		$P = -\frac{1}{x}$, $Q = 1$	½	
		$IF = e^{\int P dx} = e^{\int -\frac{1}{x}dx} = \frac{1}{x}$	1	
		Solution is $\frac{y}{x} = \log\lvert x\rvert + C$	1	
		Remarks: 1) For integrating factor, give ½ score 2) For solution $y \cdot IF = \int Q \cdot IF\,dx$, give 1 score		
16	(i)	$\begin{vmatrix} x-1 & y+1 & z-0 \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$	2	
	(ii)	The equation can be simplified as $2x+3y-3z=5$		4
		$\therefore \frac{x}{5/2} + \frac{y}{5/3} + \frac{z}{-5/3} = 1$	1	
		$\therefore a = 5/2 \quad b = 5/3 \quad c = -5/3$	1	
		Remarks: 1) For writing the formula $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$	1	
		2) For writing $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, give 1 score		

Q No	Sub Q	Answer key / Value Points	Score	Total Score
17		Let x = no. of bottles of A y = no. of bottles of B	½	
		∴ LPP is Max. $Z = x+y$, subject to	1	4
		$20x+40y \leq 3000$	1	
		$30x+10y \leq 5000$	1	
		$x, y \geq 0$	½	
18	(i)	A and B are independent		
		$\therefore P(A/B) = P(A)$, $P(B/A) = P(B)$	1	
		$P(A/B) \cdot P(B/A) = \frac{2}{10} \times \frac{4}{10} = \frac{2}{25}$	1	
	(ii)	$P(A \cap B) = P(A) \cdot P(B) = \frac{2}{25}$	½	4
		$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$	½	
		$= \frac{2}{10} + \frac{4}{10} - \frac{2}{25} = \frac{13}{25}$	1	
19.	(i)	Let $3+4x_1 = 3+4x_2$	1	
		Then $x_1 = x_2$ ∴ f is one-one	1	
	(ii)	Put $x_1 = 1$, $x_2 = -1$	½	
		Then $g(x_1) = 1$, $g(x_2) = 1$.	½	6
		∴ g is many one		
	(iii)	$fog(x) = f(g(x))$	½	
		$= 3+4x^2$	1	
		$gof(x) = g(f(x))$	½	
		$= (3+4x)^2$	1	

Remarks: 1) For the concept of one-one, give 1 score
2) For the concept of many-one, give ½ score
3) For the concept of composite function, give 1 score.

Q No	Sub Q	Value Points / Answer key	Score	Total Score
20	(i)	$\|A\| = \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = 2-20 = -18$	1	
	(ii)	$3A = \begin{bmatrix} 6 & 12 \\ 15 & 3 \end{bmatrix}$	1	
		$\|3A\| = 18 - 180 = -162$	1	6
		$9\|A\| = 9 \times -18 = -162$	1	
	(iii)	$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$	1	
		$\therefore \; 2x^2 - 24 = 2-20$	1	
		$2x^2 = 6, \quad x = \pm\sqrt{3}$		
21	(i)	$y = \cos(x^2)$		
		$\frac{dy}{dx} = -\sin(x^2) \times 2x = -2x\sin(x^2)$	2	
	(ii)	$y = e^x(x^2-1)$		
		$\frac{dy}{dx} = e^x(2x) + (x^2-1) \times e^x$	1/2	
		$= 2xe^x + e^x(x^2-1)$		6
		$= 2xe^x + y$	1/2	
		$\frac{d^2y}{dx^2} = 2xe^x + 2xe^x + \frac{dy}{dx}$	1	
		$= 2e^x(x+1) + \frac{dy}{dx}$	1	
		$= 2e^x \frac{(x^2-1)}{x-1} + \frac{dy}{dx}$		
		$= \frac{2y}{x-1} + \frac{dy}{dx}$	1	
		Remarks: Finding $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and proving the relation by substituting in it, give full score		

Q No	Sub Q	Answer key / Value points	Score	Total score
22	(i)	Cofactor matrix $= \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}$	1	
		$\lvert A \rvert = 10$	1	
		$\text{adj}(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	1	6
		$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	1	
	(ii)	$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$	1	
		$X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$	1	
		Remarks: 1) Any 6 correct values in cofactor matrix, give 1 score 2) For $A^{-1} = \frac{1}{\lvert A \rvert}\text{adj } A$, give 1 score 3) For $\text{adj } A = (\text{cofactor } A)'$, give $\frac{1}{2}$ score 4) For writing $AX = B$ and $X = A^{-1}B$, give $\frac{1}{2}$ score		
23	(i)	$\bar{a} + \bar{b} = 3\hat{i} + 3\hat{j} + 2\hat{k}$	1	
		$\bar{a} - \bar{b} = -\hat{i} + \hat{j} + 4\hat{k}$	1	
	(ii)	$(\bar{a}+\bar{b}) \times (\bar{a}-\bar{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 2 \\ -1 & 1 & 4 \end{vmatrix}$	1	6
		$= 10\hat{i} - 14\hat{j} + 6\hat{k}$	1	
	(iii)	Unit vector $= \frac{10\hat{i} - 14\hat{j} + 6\hat{k}}{\sqrt{100 + 196 + 36}} = \frac{10\hat{i} - 14\hat{j} + 6\hat{k}}{\sqrt{332}}$	1+1	
		Remarks: 1) For concept and formula for cross product of two vectors, give 1 score 2) For concept and formula for unit vector, give 1 score.		

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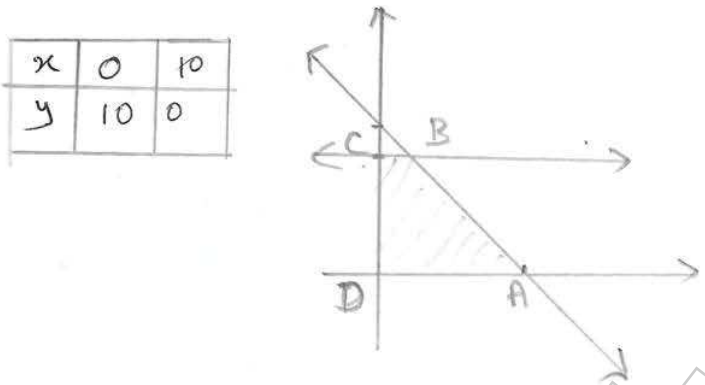
24	(i)		4.	6
	(ii)	The corner points are A(10,0), B(2,8), C(0,8), D(0,0)	1	
		Value of Z at A = 30 , which is maximum	1	
		Value of Z at B = 22		
		Value of Z at C = 16		
		Value of z at D = 0		
		Remarks: 1) For drawing x axis and y axis ½ scores each 2) For drawing x+y=10, give 2 scores		
25	(i)	$k + k + 0.1 + 0 + 2k + 2k + 0.2 + k = 1$	½	
		$\therefore 7k + 0.3 = 1$, and $k = \frac{0.7}{7} = 0.1$	½	
	(ii)	(a) $P(x \leq 2) = 2k + 0.1 = 0.3$	1	
		(b) $P(x > 3) = 3k + 0.2 = 0.5$	1	6
	(iii)	x_i : 0 1 2 3 4 5 6 P_i : 0.1 0.2 0 0.2 0.2 0.2 0.1 $x_i P_i$: 0 0.2 0 0.6 0.8 1 0.6	2	
		Mean $= \sum x_i P_i = 3.2$	1	
		Remarks: 1) For the concept $\sum P_i = 1$, give ½ score 2) For the formula mean $= \sum x_i p_i$, give ½ score		

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