SECOND YEAR HIGHER SECONDARY MODEL EXAMINATION, FEBRUARY 2024

Part III

MATHEMATICS (SCIENCE)

(Maximum 60 scores)

Answers

1. $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$

$A' = \begin{bmatrix} 3 & 0 & -2 \\ 2 & 4 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

$A + A' = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix}$

$P' = \frac{1}{2}\begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix} = P$, is symmetric.

$A - A' = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$

$Q' = \frac{1}{2}\begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix} = -Q$, is skew-symmetric.

Now $P + Q = \frac{1}{2}\begin{bmatrix} 6 & 2 & -1 \\ 2 & 8 & 0 \\ -1 & 0 & 6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 3 & 4 & 2 \\ 0 & 8 & -2 \\ -4 & 2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & -1 \\ -2 & 1 & 3 \end{bmatrix}$

$= A$, a square matrix. Hence verified.

2) i) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

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ii) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(2\pi - \frac{5\pi}{6}\right) = \cos^{-1}\cos\frac{5\pi}{6} = \frac{5\pi}{6} \in [0, \pi]$

3) $\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x^2-1) = 2^2 - 1 = 3$

$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(2x-1) = 2(2) - 1 = 3$

$\text{LHL} = \text{RHL}$

Now $\lim_{x\to 2} f(x) = 4$

Since $\text{LHL} = \text{RHL} \neq f(2)$

$\therefore$ $f(x)$ is not continuous at $x = 2$

4. i) $f(x)$ is decreasing in the internal $(-\infty, 1)$ and decreasing in the internal $(3, \infty)$

ii) Local maxima at $x = 1$ and local minima at $x = 3$

5. $I = \int \frac{x-1}{x^2-4x+5} dx$

Let $\frac{x-1}{x^2-4x+5} = \frac{x-1}{(x+1)(x-5)} = \frac{A}{x+1} + \frac{B}{X-5}$ ………… (1)

$x - 1 = A(x-5) + B(x+1)$

Put $x = 5$

$5 - 1 = A(0) + B(6)$

$\therefore 6B = 4$

$\therefore B = \frac{4}{6} = \frac{2}{3}$

Put $x = -1$

$-1 - 1 = A(-1-5) + B(0)$

$-2 = -6A$

$A = \frac{-2}{-6} = \frac{1}{3}$

in (1) $\frac{x-1}{(x+1)(x-5)} = \frac{1/3}{x+1} + \frac{2/3}{x-5}$

$\therefore I = \frac{1}{3}\int \frac{dx}{x+1} + \frac{2}{3}\int \frac{dx}{x-5}$

$= \frac{1}{3}\log|x+1| + \frac{2}{3}|x-5| + C$

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6. i) $\overrightarrow{AB} = (4-1)\hat{\imath} + (5-5)\hat{\jmath} + (7-3)\hat{k}$

$= 3\,\hat{\imath} + 0\hat{\jmath} + 4\hat{k}$

ii) $|\overrightarrow{AB}| = \sqrt{9+0+16} = \sqrt{25} = 5$

Unit vector in the direction of $\overrightarrow{AB} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

$= \frac{3\hat{\imath}+0\hat{\jmath}+4\hat{k}}{5}$

$= \frac{3}{5}\hat{\imath} + \frac{0}{5}\hat{\jmath} + \frac{4}{5}\hat{k}$

iii) c) $-5\hat{\jmath}$

7) i) f is many one and onto

ii) $f(x_1) = f(x_2)$

$4x_1 = 4x_2$

$x_1 = x_2$

$\therefore f$ is one-one.

Now $f(x) = 4x$

$y = 4x$

$\therefore \quad x = \frac{y}{4}$

$\therefore f\left(\frac{y}{4}\right) = 4 \times \frac{y}{4} = y$

$\therefore f$ is onto

$\therefore f$ is bijective

$\therefore f$ is invertible

$\therefore f^{-1} = \frac{y}{4}$

$\therefore f^{-1}(x) = \frac{x}{4}$

8. Let E: the number 5 appears atleast once.

F: Sum=9

$E = \{(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (5,1), (5,2)(5,3), (5,4), (5,6)\}$

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$F = \{(5,4),(4,5),(3,6),(6,3)\}$

$P(E \cap F) = \frac{2}{36}$

$P(F) = \frac{4}{36}$

$\therefore P(E/F) = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{2}{4} = \frac{1}{2}$

9. i) $(1,1),(2,2)$ and $(3,3) \in R$

$\therefore$ R is reflexive

$(1,3) \in R, (3,1) \in R \implies (1,1) \in R$

$\therefore$ R is transitive.

ii) Equivalence classes: $\{1,3\},\{2\},\{3,1\}$

10) i) $x - 1 = 2 \implies x = 2 + 1 = 3$

$-2 - 2 = 2y \Rightarrow 2y = -4 \Rightarrow y = \frac{-4}{2} = -2$

ii) $AB = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 5 \times 2 & 5 \times -1 & 5 \times -3 \\ -2 \times 2 & -2 \times -1 & -2 \times -3 \\ 3 \times 2 & 3 \times -1 & 3 \times -3 \end{bmatrix} =$

$= \begin{bmatrix} 10 & -5 & -15 \\ -4 & 2 & 6 \\ 6 & -3 & -9 \end{bmatrix}$

11. i) Let the side of the square be $x\ cm$

$V = x^3$

$\frac{dV}{dt} = 3x^2 . \frac{dx}{dt}$

$24 = 3 \times 6^2 \times \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = \frac{24}{3 \times 6 \times 6} = \frac{2}{9} cm/s$

Surface area, $A = 6x^2$

$\frac{dA}{dt} = 6.2x . \frac{dx}{dt}$

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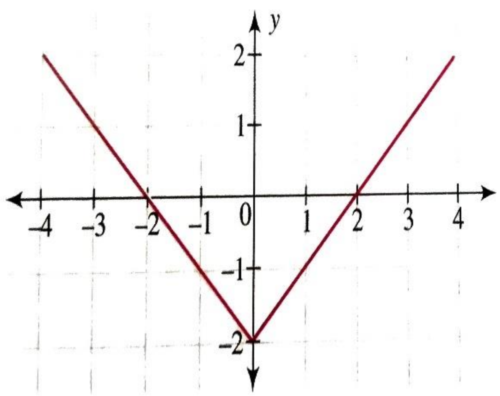
$$= 12 \times 6 \times \frac{2}{9} = 4 \times 2 \times 2 = 16\, cm^2/s$$

ii)



From the graph it is clear that local minimum value of the function is -2

12. $\frac{x^2}{25} + \frac{y^2}{9} = 1 \Longrightarrow \frac{y^2}{9} = 1 - \frac{x^2}{25} \Longrightarrow \frac{y^2}{9} = \frac{25-x^2}{25}$

$\Longrightarrow y^2 = \frac{9}{25}(25 - x^2) \Longrightarrow y = \frac{3}{5}\sqrt{25 - x^2}$

Area of the ellipse $= 4\int_0^5 \frac{3}{5}\sqrt{5^2 - x^2}\, dx$

$$= 4 \times \frac{3}{5}\int_0^5 \sqrt{5^2 - x^2}\, dx$$

$$= 4 \times \frac{3}{5}\left[\frac{x}{5}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= 4 \times \frac{3}{5}\left[\frac{5}{5}\sqrt{5^2 - 5^2} + \frac{5^2}{2}\sin^{-1}\left(\frac{5}{5}\right) - \left\{\frac{5}{5}\sqrt{5^2 - 5^2} + \frac{5^2}{2}\sin^{-1}\left(\frac{5}{5}\right)\right\}\right]$$

$$= \frac{12}{5}\left[0 + \frac{25}{2}\sin^{-1}(1) - \left\{0 + \frac{0^2}{2}\sin^{-1}\left(\frac{0}{5}\right)\right\}\right]$$

$$= \frac{12}{5}\left[\frac{25}{2} \times \frac{\pi}{2} - 0\right] = \frac{12}{5} \times \frac{25\pi}{4} = 3 \times 5\pi = 15\pi \text{ square units.}$$

13. i) $y = e^x + 1$

ii) $\frac{dy}{dx} = \frac{\sqrt{9-y^2}}{x} \Longrightarrow \frac{dy}{\sqrt{9-y^2}} = \frac{dx}{x}$ is in variable separable

$\int \frac{dy}{\sqrt{9-y^2}} = \int \frac{dx}{x}$

$\int \frac{dy}{\sqrt{3^2-y^2}} = \int \frac{dx}{x}$

$\sin^{-1}\left(\frac{y}{3}\right) = \log|x| + c$

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14 i) $\vec{a} = 2\hat{\imath} + \hat{\jmath}\,, \vec{b} = 2\vec{a} = 4\hat{\imath} + 2\hat{\jmath}$

$\vec{a}.\vec{b} = (2)(4) + (1)(2) = 10$

$|\vec{b}| = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\,.\,\vec{b}}{|\vec{b}|} = \frac{10}{2\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$

ii) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix} = (1+4)\hat{\imath} - (3-4)\hat{\jmath} + (-3-1)\hat{k}$

$= 5\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\therefore$ Area of parallelogram= $|\vec{a} \times \vec{b}| = \sqrt{25+1+16} = \sqrt{42}$ square units.

15. $Let\ A = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2-1 & 1-1 & -1-0 \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & -1 \\ 2 & -1 & 2 \\ 3 & -5 & 2 \end{vmatrix} = 1[-2+10] - 0 - 1[-10+3] = 8 - 1(-7) = 8 + 7 = 15$

$B = \sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}$

$B = \sqrt{(-2+10)^2 + (6-4)^2 + (-10+3)^2} = \sqrt{64+4+49} = \sqrt{117}$

$\therefore shortest\ distance\ between\ the\ lines = \left|\frac{A}{B}\right| = \frac{15}{\sqrt{117}}$ units.

16. i) $P(A' \cap B') = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A)P(B)]$

$= 1 - [0.4 + 0.5 - 0.4 \times 0.5] = 1 - [0.9 - 0.2]$

$= 1 - 0.7 = 0.3$

ii) $E_1: die\ shows\ 5$

$E_2: die\ not\ shows\ 5$

$P(E_1) = \frac{1}{6}$; $P(E_2) = \frac{5}{6}$

Let $E: A\ man\ reports\ that\ it\ is\ 5$

$P(E/E_1) = \frac{4}{5}$; $P(E/E_2) = \frac{1}{5}$

Required probability, $P(E_1/E) = \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{1}{5}\times\frac{5}{6}} = \frac{1\times4}{1\times4+1\times5} = \frac{4}{9}$

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17. $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & -3 & 4 \end{vmatrix} = 1(4+3) - 2(8-1) + 1(-6-1)$

$= 1(7) - 2(7) + 1(-7) = 7 - 14 - 7 = -14 \neq 0$

Cofactors:

$A_{11} = 7 \quad ; A_{12} = -7 \quad ; A_{13} = -7$

$A_{21} = -11 \quad ; A_{22} = 3 \quad ; A_{23} = 5$

$A_{31} = 1 \quad ; A_{32} = 1 \quad ; A_{33} = -3$

Cofactor matrix of $A = \begin{bmatrix} 7 & -7 & -7 \\ -11 & 3 & 5 \\ 1 & 1 & -3 \end{bmatrix}$

Adjoint matrix of $A = \begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}.adj\ A = \frac{1}{-14}\begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}$

$\therefore X = A^{-1}B = \frac{1}{-14}\begin{bmatrix} 7 & -11 & 1 \\ -7 & 3 & 1 \\ -7 & 5 & -3 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 3 \end{bmatrix} = -\frac{1}{14}\begin{bmatrix} 126-55+3 \\ -126+15+3 \\ -126+25-9 \end{bmatrix} = -\frac{1}{14}\begin{bmatrix} 74 \\ -108 \\ -110 \end{bmatrix}$

Hence, $x = -\frac{74}{14}$; $y = \frac{108}{14}$ $and\ z = \frac{110}{14}$

18. i) $y = \sqrt{sinx}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{sinx}} \times cosx$

ii) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-asint}{a(1-cost)} = \frac{-2\sin\frac{t}{2}\cos\frac{t}{2}}{2\sin^2\frac{t}{2}} = -\cot\frac{t}{2}$

iii) $y = x^x$

$logy = xlogx$

$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + logx \times 1$

$\frac{dy}{dx} = y(1 + logx) = x^x(1 + logx)$

19. i) $I = \int e^x sinx\,dx$

Using integrating by parts, we have

$= e^x(-cosx) - \int e^x(-cosx)dx$

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$= -e^x cosx + \int e^x cosx\,dx$

$= -e^x cosx + e^x(sinx) - \int e^x sinx\,dx$

$= -e^x cosx + e^x(sinx) - I$

$2I = e^x(sinx - cosx)$

$\therefore\ I = \frac{1}{2}e^x(sinx - cosx) + c$

ii) $I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos^3 x + \sin^3 x}dx$ ....(1)

We have $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\therefore\ I = \int_0^{\frac{\pi}{2}} \frac{\sin^3\left(\frac{\pi}{2}-x\right)}{\cos^3\left(\frac{\pi}{2}-x\right)+\sin^3\left(\frac{\pi}{2}-x\right)}dx$

$= \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sin^3 x + \cos^3 x}dx$ .... (2)

(1) + (2) we have,

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos^3 x + \sin^3 x}dx + \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\sin^3 x + \cos^3 x}dx$

$2I = \int_0^{\frac{\pi}{2}} \left[\frac{\sin^3 x}{\sin^3 x + \cos^3 x} + \frac{\cos^3 x}{\sin^3 x + \cos^3 x}\right]dx$

$2I = \int_0^{\frac{\pi}{2}} \left[\frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}\right]dx = \int_0^{\frac{\pi}{2}} 1\,dx = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$

20. $3x + 4y = 60$

x	20	0
y	0	15

$x + y = 10$

x	10	0
y	0	10

$x - 3y = -6$

x	-6	0
y	0	2

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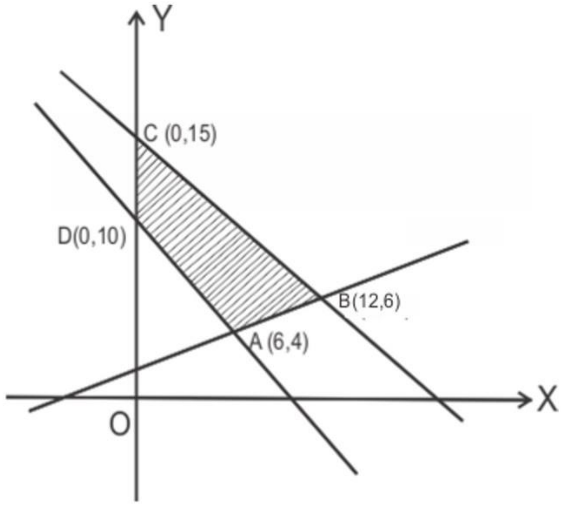

The corner points are $A(6,4), B(12,6), C(0,15)\ and\ D(0,10)$

Points	$Z = 3x + 7y$
$A(6,4))$	$Z = 3(6) + 7(4) = 46$
$B(12,6)$	$Z = 3(12) + 7(6) = 72$
$C(0,15)$	$Z = 3(0) + 7(15) = 105$
$D(0,10)$	$Z = 3(0) + 7(10) = 70$

$\therefore Z_{max} = 105\ \ at\ C(0,15)$

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