

**SECOND YEAR HIGHER SECONDARY  
MODEL EXAMINATION , MARCH 2022****Part I****A. Answer any five questions (5 x 1 = 5)**

1.  $x$

2.  $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x) = \cos \frac{\pi}{2} = 0$

3. c)  $k^3|A|$

4. Area of circle =  $\pi r^2$

$$\frac{dA}{dr} = 2\pi r$$

$$= 12\pi$$

5.  $f(x) = \sin x$

$$f(-x) = \sin(-x) = -\sin x$$

$f(x)$  is an odd function.

$$\text{Therefore } \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx = 0$$

6. Degree = 1

$$7. \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 8 & 4 & 12 \end{vmatrix} = 4 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 2 & 1 & 3 \end{vmatrix} = 0$$

8.  $\vec{r} = \lambda (2\hat{i} + 4\hat{j} + 2\hat{k})$

9.  $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$

**B. Answer all questions (4 x 1 = 4)**

10.  $\sin x = \frac{1}{2}$

Principal values is  $\frac{\pi}{6}$

11. 0

12.  $y = e^{\log x}$

that is,  $y = x$

$$\frac{dy}{dx} = 1$$

13. 1

**Part II****A. Answer any two questions (2 x 2 = 4)**

14.  $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$a_{11}=1, a_{12}=0, a_{21}=3, a_{22}=2$$

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

15.  $f(x) = x^2 - 4x + 6$

$$f'(x) = 2x - 4$$

$$f'(x) = 0 \Rightarrow 2x - 4 = 0$$

$$x = 2$$

2 divides  $\mathbb{R}$  into  $(-\infty, 2)$  and  $(2, \infty)$

In  $(2, \infty)$ ,  $f'(x) > 0$ ,

$f(x)$  is strictly increasing in  $(2, \infty)$

16.  $\frac{dy}{dx} = 3x^2$

slope, at (1,1) = 3

slope of normal =  $-\frac{1}{3}$

Equation of normal,

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$\text{ie, } x + 3y - 4 = 0$$

17. Let  $y = mx$

$$\frac{dy}{dx} = m$$

$$\text{So, } y = \frac{dy}{dx} x$$

**B. Answer any two questions (2 x 2 = 4)**

18.  $x - y = \pi$

differentiating w.r.t  $x$

$$1 - \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 1$$



19.  $\frac{dy}{dx} = 1 + \frac{y}{x}$

$$\frac{dy}{dx} - \frac{y}{x} = 1$$

$$P = -\frac{1}{x}, Q = 1$$

$$I . F = e^{\int -\frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$$

General solution is,

$$y . IF = \int Q . IF dx + C$$

$$y . \frac{1}{x} = \int \frac{1}{x} dx + C$$

$$\frac{y}{x} = \log |x| + C$$

20. Vectors are coplanar if  $[\vec{a} \vec{b} \vec{c}] = 0$

$$\begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(15 - 12) + 2(-10 + 4) + 3(6 - 3)$$

$$= 3 - 12 + 9 = 0$$

Therefore the vectors are coplanar.

### Part III

**A. Answer any three questions (3 x 3 = 9)**

21. (1,1), (2,2), (3,3) ∈ R

therefore R is reflexive.

(1,2) ∈ R but (2,1) ∉ R

therefore R is not reflexive.

(1,2), (2,3) ∈ R but (1,3) ∉ R

Therefore R is not transitive.

22.  $A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$kA - 2I = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k \end{bmatrix}$$

$$A^2 = kA - 2I$$

$$\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k \end{bmatrix}$$

$$\text{ie, } 3k - 2 = 3$$

$$k = 1$$

23. Area of parallelogram =  $|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(1+4) - \hat{j}(3-4) + \hat{k}(-3-1)$$

$$= 5\hat{i} + \hat{j} - 4\hat{k}$$

$$\text{Area} = \sqrt{25 + 1 + 16} = \sqrt{42}$$

24. Probability of solving A, P(A) =  $\frac{1}{2}$

Probability of solving B, P(B) =  $\frac{1}{3}$

A and B are independent events.

$$P(A') = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(B') = 1 - \frac{1}{3} = \frac{2}{3}$$

Probability of exactly one of them solves the

$$\text{problem} = P(A \cap B') + P(A' \cap B)$$

$$= P(A) \cdot P(B') + P(A') \cdot P(B)$$

$$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$$

**B. Answer any two questions (2 x 3 = 6)**

25.

∧	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

26.  $A = I A$

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow \frac{R_2}{-5}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix} A$$

$$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

27. By definition

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} h [f(a) + f(a+h) + \dots + f(a+(n-1)h)]$$

where  $h = \frac{b-a}{n} \rightarrow 0$  as  $n \rightarrow \infty$

Here  $h = \frac{2-0}{n} = \frac{2}{n}$

$$f(a) = f(0) = 0$$

$$f(a+h) = (h)^2$$

$$f(a+2h) = (2h)^2$$

$$f(a+3h) = (3h)^2$$

.....

$$f(a+(n-1)h) = ((n-1)h)^2$$

So,

$$\begin{aligned} \int_0^2 x^2 dx &= \lim_{n \rightarrow \infty} \frac{2}{n} [h^2 + 2^2 \cdot h^2 + 3^2 \cdot h^2 + \dots + (n-1)^2 \cdot h^2] \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} [h^2 (1^2 + 2^2 + 3^2 + \dots + (n-1)^2)] \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left(\frac{2}{n}\right)^2 \frac{(n-1)n(2n-1)}{6} \\ &= \lim_{n \rightarrow \infty} \frac{4}{3} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) \\ &= \frac{4}{3} \times 1 \times 2 = \frac{8}{3} \end{aligned}$$

**Part IV**

**A. Answer any three questions ( 3 x 4 = 12 )**

28. (i)  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \left( \frac{\frac{1}{2} + \frac{2}{11}}{1 - \frac{1}{2} \times \frac{2}{11}} \right)$

$$= \tan^{-1} \left( \frac{\frac{15}{22}}{\frac{20}{22}} \right)$$

$$= \tan^{-1} \left( \frac{3}{4} \right)$$

(ii) Put  $x = \cos \theta$ , then  $\theta = \cos^{-1} x$

$$\begin{aligned} \cos^{-1}(4x^3 - 3x) &= \cos^{-1}(4 \cos^3 \theta - 3 \cos \theta) \\ &= \cos^{-1}(\cos 3\theta) \\ &= 3\theta \\ &= 3 \cos^{-1} x \end{aligned}$$

29.  $f(x)$  is continuous on  $[2,4]$

$f(x)$  is differentiable on  $(2,4)$

$$f(a) = f(2) = 2^2 = 4$$

$$f(b) = f(4) = 4^2 = 16$$

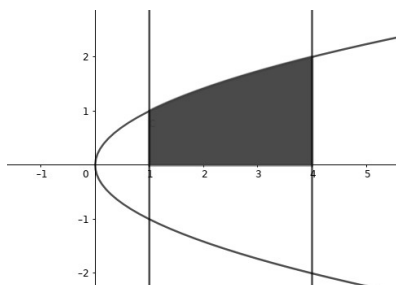
$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{16 - 4}{4 - 2} = \frac{12}{2} = 6$$

But  $f'(c) = 2c$

$$2c = 6 \Rightarrow c = 3 \in (2,4) \text{ . Hence verified.}$$



30.



Area =

$$\int_0^4 y \, dx = \int_0^4 \sqrt{x} \, dx = \left[ \frac{x^{3/2}}{3/2} \right]_0^4 = \frac{2}{3} \left( 4^{3/2} - 0 \right) = \frac{16}{3}$$

31.  $\vec{a}_1 = \hat{i} + \hat{j}$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+1+49} = \sqrt{59}$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k}) = 10$$

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{10}{\sqrt{59}}$$

**B. Answer any one question ( 1 x 4 = 4 )**

32. a)  $0.1 + k + 2k + 2k + k = 1$

$$0.1 + 6k = 1$$

$$k = 0.15$$

b)  $P(x < 3) = P(x=0) + P(x=1) + P(x=2)$

$$= 0.1 + k + 2k$$

$$= 0.1 + 3 \times 0.15$$

$$= 0.55$$

33. Cartesian form is  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$$

$$\frac{6x + 4y + 3z}{12} = 1$$

ie  $6x + 4y + 3z = 12$

Vector form is  $\vec{r} \cdot \hat{n} = d$

$$\vec{r} \cdot (6\hat{i} + 4\hat{j} + 3\hat{k}) = 12$$

**Part V**

**Answer any two questions ( 2 x 6 = 12 )**

34.  $A X = B$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$



$$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} = -17$$

Co-factor matrix =  $\begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \frac{1}{-17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$= \frac{1}{-17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

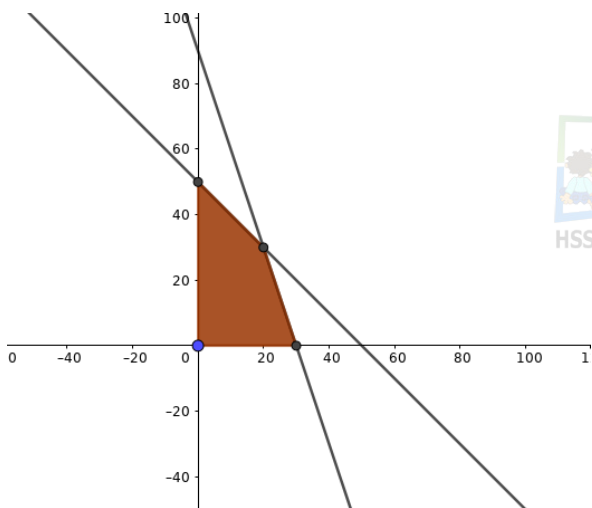
$x = 1, y = 2, z = 3$

35.  $x + y = 50$

x	y
0	50
50	0

$3x + y = 90$

x	y
0	50
50	0



Point	$Z = 4x + y$
(0,0)	0
(30,0)	120
(20, 30)	110
(0,50)	50

Maximum of  $Z$  is 120 at (30,0).

36. i) put  $t = \tan^{-1} x$ , then  $dt = \frac{1}{1+x^2} dx$

$$\begin{aligned} \int \frac{\sin(\tan^{-1} x)}{1+x^2} dx &= \int \sin t dt \\ &= -\cos t \\ &= -\cos(\tan^{-1} x) \end{aligned}$$

ii) Let  $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Then

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)} + \sqrt{\cos(\frac{\pi}{2}-x)}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \end{aligned}$$

Adding them, we get

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \\ &= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \\ &= \int_0^{\frac{\pi}{2}} 1 dx = \frac{\pi}{2} \\ I &= \frac{\pi}{4} \end{aligned}$$

iii)  $\int \frac{dx}{x^2-16} = \int \frac{dx}{x^2-4^2}$

$$\begin{aligned} &= \frac{1}{2 \times 4} \log \left| \frac{x-4}{x+4} \right| + c \\ &= \frac{1}{8} \log \left| \frac{x-4}{x+4} \right| + c \end{aligned}$$

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